Test 1 - MTH 1400 Online

Dr. Adam Graham-Squire, Summer 2018

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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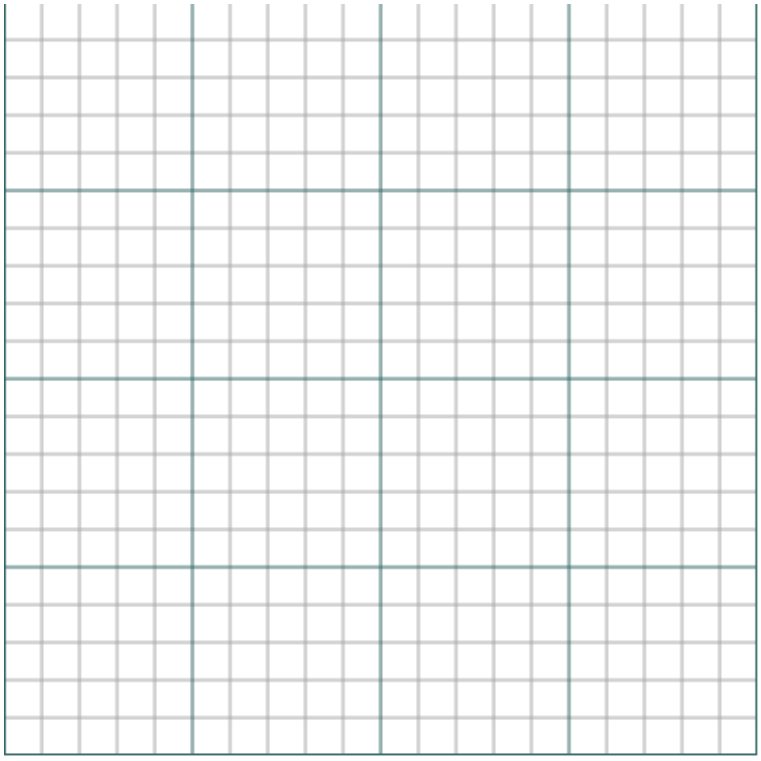
(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. No calculators are allowed on the first 7 questions of the test. Calculators ARE allowed on the last 5 questions, however you should still show all of your work. You will initially receive the entire test, and you will NOT be allowed a calculator. Once you have finished everything you can without a calculator, you should turn in the first part of the test (the first 8 questions and extra credit question) to the proctor. The proctor can then give you your calculator and you can finish the remaining questions. You are not allowed to go back to the No Calculator portion once you have been given your calculator.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 12. Total Points = 60.

1. (5 points) Duke Energy charges its electric customers a base rate of \$5 a month, plus 10¢ per kilowatt-hour (kWh) for the first 100 kWh used and 4¢ per kWh for all usage above 100kWh. Suppose a customer uses x kWh of electricity in one month.

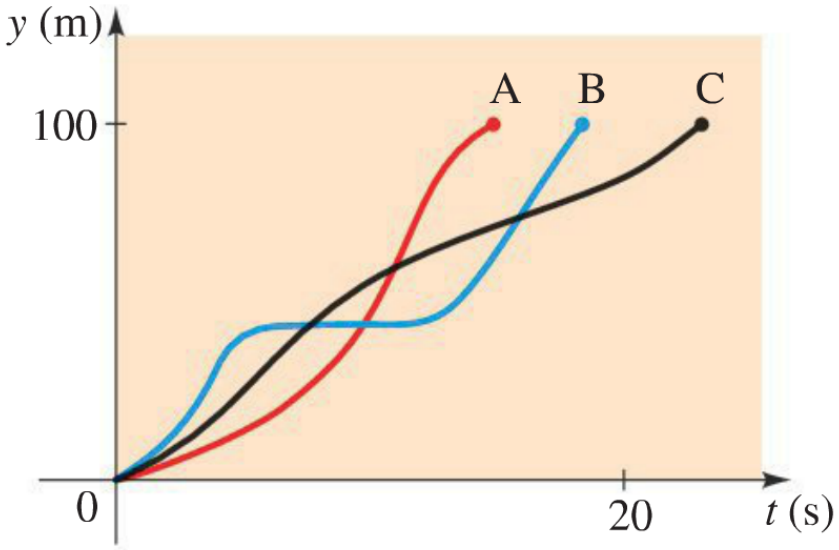
   (a) Sketch a graph of the monthly cost C as a function of x (that is, put cost on the y-axis and x on the x-axis) for x-values $0 \leq x \leq 400$.



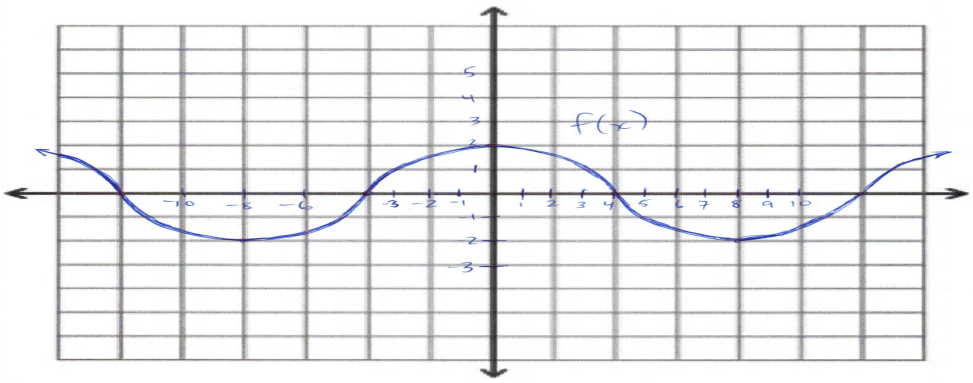
   (b) Express the monthly cost C as a piecewise defined function of x. That is, fill in the blanks:

$$C(x) = \begin{cases} \_\_\_\_\_ \text{ if } \_\_\_\_\_ \\ \\ \_\_\_\_\_ \text{ if } \_\_\_\_\_ \end{cases}$$

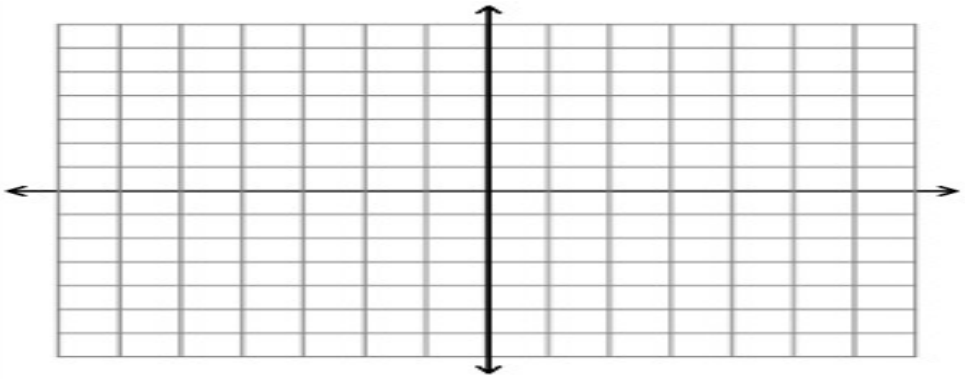
2. (5 points) Three runners compete in a 100-meter hurdle race. The graph shows the distance run (in meters) as a function of time (in seconds) for each runner. Describe in words what the graph tells you about this race. In particular, answer these questions: Who won the race? Did each runner finish the race? What happened to Runner B? Was the winner of the race in the lead the whole time?



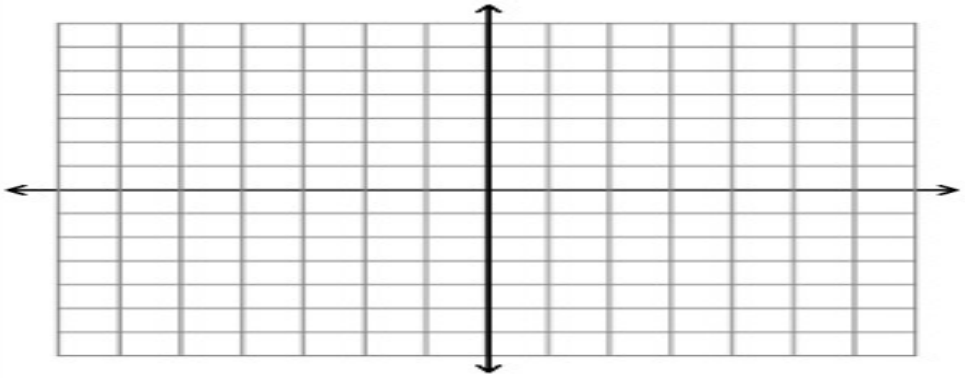
3. (5 points) Use the graph of $f(x)$ to sketch the graphs below:

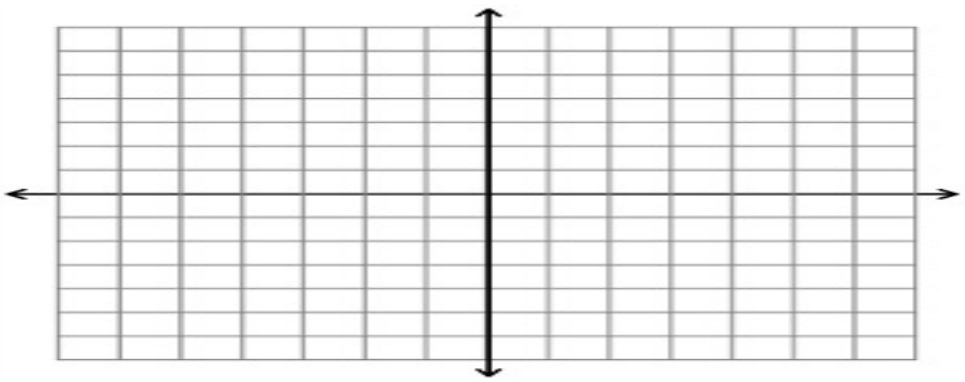


(a) $f(2x)$:



(b) $3f(x+5)-1$ (there are two graphs so you can do a rough and final sketch.):



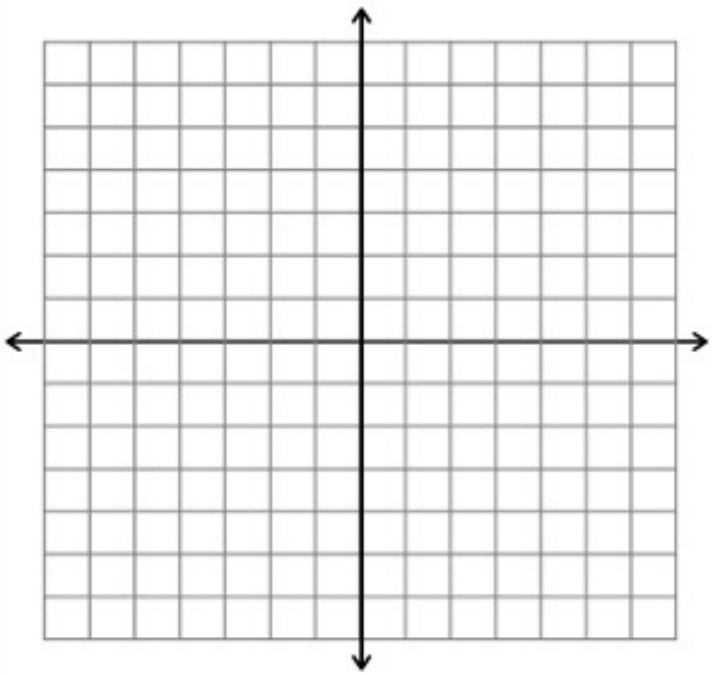


4. (5 points) Calculate the average rate of change for the function $f(x) = \dfrac{2}{x}$ between $t = 3$ and $t = 3 + h$. Simplify your answer as much as possible.

5. (5 points) Let $P(x) = -2x^4 - x^3 + 10x^2$.

   (a) Factor to find the zeroes of P.

   (b) Sketch a graph of P. Explain (briefly) in words how you know where the graph goes.



6. (5 points) Find all vertical and horizontal asymptotes of $y = \dfrac{3x+6}{x^2-3x-18}$, if any exist. Explain/show your work.

7. (3 points) Find all solutions to the equation $\log_2(25 - x^2) = 4$.

Extra Credit (2 points): Simplify the following expression as much as possible (logarithm rules may be helpful):

$$\log_{0.5} 8 - 3 \log_{0.5} 4$$

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Calculator Okay portion Name:_____

•You can work on these problems without a calculator, but when you want/need to use a calculator, you must turn in the No Calculator portion of the test to the proctor.

•If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

8. (5 points) (a) Find all solutions to the equation $x^2 + 2x + 2 = 0$.

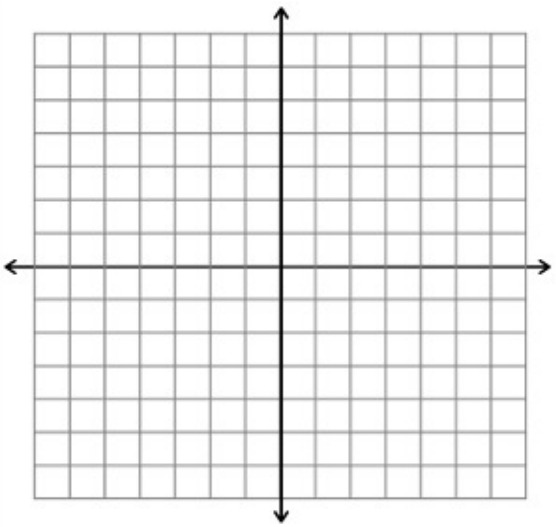
   (b) Substitute one of your solutions back into the original equation to confirm that it is in fact a solution.

9. (5 points) Let $g(x) = \dfrac{x^3 - 5}{8}$.

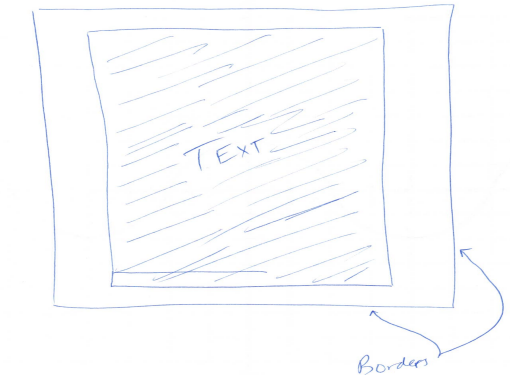
   (a) Find $g^{-1}(x)$. Show your work.

   (b) Graph both $g(x)$ and $g^{-1}(x)$ on the coordinate axes below. How can you tell that they are inverse functions from the graph?

   (c) Compose the functions g and g^{-1} to confirm that they are inverses of one another.



10. (10 points) Ruby needs to make a poster to display her research at a math conference. The poster will have a box of text in the middle, and then a white border around the outside. She needs to have 16 square feet of space for the text, and that area can have any rectangular dimensions. Ruby also needs to have a 1 foot border at the left and right of the text box, and a 0.5 foot border at the top and bottom of the text box. Use the diagram below of the poster to help answer the questions.



(a) If Ruby makes the interior text box 4 feet by 4 feet, what will be the overall area (in square feet) of the poster (including the borders)?

(b) If she makes the interior text box 2 feet by 8 feet, what will be the overall area (in square feet) of the poster (including the borders)?

(c) If she makes the interior text box 8 feet by 2 feet, what will be the overall area (in square feet) of the poster (including the borders)?

(d) If the width of the text box is x, the height must be $16/x$. Explain why this must be the case.

(e) Using x for the width of the text box is x and $16/x$ for the height, write an equation for the overall area of the poster.

(f) Graph your equation from part 10e to calculate the minimum size (in amount of area) Ruby can make the poster (Note that she does NOT have to make the dimensions a whole number. Decimal numbers are fine, round to nearest 0.01 ft^2). Does your answer make sense? Explain why or why not.

Left blank if you need more room for question 10.

11. (5 points) Let

$$P(t) = \frac{1800}{2 + 7e^{-0.3t}}$$

be the population function for a group of rabbits on an island, where P is the number of rabbits and t is in years. Round down all answers, if necessary (I don't believe in fractional bunnies).

(a) What is the original population of the rabbits?

(b) What will the population be after 10 years? 20 years?

(c) What will the stable population be in the long run? Explain how you can figure out the answer *without* using a calculator. It is fine to check your answer by using a calculator, but for full credit you must explain how it can be found without using a calculator as well.

12. (2 points) Find all solutions to the equation

$$3000 = 500e^{0.23x}.$$

Round answer(s) to nearest 0.01.